Exercise 14

Find the limit.

$$\lim_{x\to-\infty}\frac{\sqrt{x^2-9}}{2x-6}$$

Solution

Start by making the limit go to infinity instead by making the substitution, $u=-x$. Then as $x\to-\infty$, $u\to\infty$.

$$\begin{aligned}\lim_{x\to-\infty}\frac{\sqrt{x^2-9}}{2x-6} &= \lim_{u\to\infty}\frac{\sqrt{(-u)^2-9}}{2(-u)-6}\\ &= \lim_{u\to\infty}\frac{\sqrt{u^2-9}}{-2u-6}\\ &= \lim_{u\to\infty}\frac{\sqrt{(u+3)(u-3)}}{-2(u+3)}\\ &= \lim_{u\to\infty}\frac{1}{-2}\sqrt{\frac{u-3}{u+3}}\end{aligned}$$

Make the substitution, $v=u+3$. Then as $u\to\infty$, so does v.

$$\begin{aligned}\lim_{x\to-\infty}\frac{\sqrt{x^2-9}}{2x-6} &= \lim_{v\to\infty}\frac{1}{-2}\sqrt{\frac{(v-3)-3}{v}}\\ &= \lim_{v\to\infty}\frac{1}{-2}\sqrt{\frac{v-6}{v}}\\ &= \lim_{v\to\infty}\frac{1}{-2}\sqrt{\frac{v}{v}-\frac{6}{v}}\\ &= \lim_{v\to\infty}\frac{1}{-2}\sqrt{1-\frac{6}{v}}\\ &= \frac{1}{-2}\sqrt{1-0}\\ &= -\frac{1}{2}\end{aligned}$$